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PERFORMANCE PREDICTION MODEL FOR ROAD-CONSTRAINED MULTIPLE TARGET TRACKING

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14. ABSTRACT

The performance of tracking systems depends on numerous factors including the scenario, operating conditions, and choice of tracker algorithms. For tracker system design, mission planning, and sensor resource management, the availability of a tracker performance model (TPM) for the standard measures of performance (MOPs) would be of high practical value. Ideally, the TPM has high computational efficiency, and is insensitive to the particular low-level details of highly complex algorithms and unimportant operating conditions. These characteristics would eliminate the need for high fidelity Monte Carlo simulations that are expensive and time consuming. In this paper, we describe a performance prediction model that generates track life distributions and other MOPs. The model employs a simplified Monte Carlo simulation that accounts for sensor orbits, sensor coverage, target dynamics. A key feature is an analytical expression that approximates the probability of correct association (PCA) among reports and tracks. The expression for the PCA that we use was developed by Mori et al. for simplified scenarios where there is a single class of targets, the noise is Gaussian, and the covariance matrices are identical for all targets. Based on heuristic considerations, we extend this result to the case of road-constrained tracking where both on-road and off-road targets are present. We investigate the validity of the proposed expression by means of Monte Carlo simulations, and present preliminary results of a validation study that compares the performance of an actual tracker with the performance predictions of our model.

15. SUBJECT TERMS tracking performance prediction, measures of performance, road-constrained tracking, probability of correct association

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Performance Prediction Model for Road-Constrained Multiple Target Tracking

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ABSTRACT

The performance of tracking systems depends on numerous factors including the scenario, operating conditions, and choice of tracker algorithms. For tracker system design, mission planning, and sensor resource management, the availability of a tracker performance model (TPM) for the standard measures of performance (MOPs) would be of high practical value. Ideally, the TPM has high computational efficiency, and is insensitive to the particular low-level details of highly complex algorithms and unimportant operating conditions. These characteristics would eliminate the need for high fidelity Monte Carlo simulations that are expensive and time consuming. In this paper, we describe a performance prediction model that generates track life distributions and other MOPs. The model employs a simplified Monte Carlo simulation that accounts for sensor orbits, sensor coverage, target dynamics. A key feature is an analytical expression that approximates the probability of correct association (PCA) among reports and tracks. The expression for the PCA that we use was developed by Mori et. al. for simplified scenarios where there is a single class of targets, the noise is Gaussian, and the covariance matrices are identical for all targets. Based on heuristic considerations, we extend this result to the case of road-constrained tracking where both on-road and off-road targets are present. We investigate the validity of the proposed expression by means of Monte Carlo simulations, and present preliminary results of a validation study that compares the performance of an actual tracker with the performance predictions of our model.[†]

Keywords: Tracking Performance Prediction, Measures of Performance, Road-constrained Tracking, Probability of Correct Association

1. INTRODUCTION

An important objective in the design and application of tracker systems is performance assessment as a function of the various design parameters and operating conditions. These studies are valuable for quantifying system trade–offs, and provide a rational objective criterion for system engineering decisions and mission planning. A key problem for system design is that in the initial stages, the algorithms may not be available to perform the required evaluation. Therefore, it is necessary to predict the tracking performance without actually implementing the tracker. There are several ways to obtain these predictions. For example, results can be extrapolated from existing trackers by analyzing the effect of novel features and new operating conditions on the overall tracking performance. The accuracy of these studies, however, will depend on the similarities between the existing system and the proposed system. Another way to predict performance is by performing high fidelity Monte Carlo simulations that represent the entire system, including sensor, tracker, and scenario conditions. This is probably the most reliable method, but it can be very costly and time consuming. It is also possible to use analytical methods. Unfortunately, the tracking problem is very complex and exact analytical expressions to obtain measures of performance do not yet exist. Notwithstanding, there have been some efforts to obtain approximations that cover some aspects of the tracking problem. Mori, Chang, and Chong[2] derived an approximate expression for the Probability of Correct Association (PCA) for the optimal data association problem. The PCA is a key parameter in the assessment of multiple target tracking systems, as incorrect associations lead to degraded track estimations and eventually to the termination of the tracks. Though derived on the basis of numerous approximations, Mori et.al.'s PCA expression is remarkably accurate for a large range of operating conditions (e.g., target and false target density, measurement noise).

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The fundamental difficulty of performance prediction for tracker systems is the dynamical nature of the problem. Time-dependence is introduced by the sensor orbit, target dynamics, changing obscuration, fluctuations in the statistical variables, etc. These changes ultimately map into tracker quantities such as the target density and the Kalman Filter (KF) innovations covariance. Mori et.al.'s PCA expression explicitly depend on these quantities (see Section 2). Therefore, the PCA changes with time, yet the analytical expression was derived for a static scenario. Also, the expression was derived for a single time frame whereas real trackers often use deferred decision logic over multiple time frames. These considerations begin to suggest the complexity of the problem. As a first approximation it would be natural to consider an average innovations covariance based on the steady state solution of the KF. However, results based on this assumption would not be very accurate. This is because the increase on the errors due to the track-report misassociations has a chain reaction effect: the first misassociation increases the magnitude of innovations covariance, and this in turn decreases the PCA inducing even more misassociations. This cascade effect eventually leads to the termination of the tracks. In addition, the initial transients when the tracks have not yet been established have a large impact on the performance of the system. Real systems mitigate this effect by starting a track only after a few detections have been successfully associated, but it is clear that a performance analysis needs to consider these effects to produce reliable results. To take into account the PCA variability and achieve improved prediction accuracy, we have developed a Tracking Performance Model (TPM) that blends Monte Carlo simulations with theoretical analyses. The latter utilize the PCA analytical expression. The TPM provides track life as well as target location error statistics. This paper describes the TPM and discusses preliminary experiments that were implemented to validate this model.

Currently, the TPM is strictly only applicable to trackers that utilize instantaneous assignment algorithms (e.g., perform association using data from a single time frame only). Trackers that utilize data from multiple time frames such as Multiple Hypothesis Tracking (MHT) algorithms and multidimensional assignment algorithms, will generally display a different statistical behavior for the PCA. Unfortunately, a closed form expression for the PCA is not yet available. In a first attempt to address the single time-frame limitation, we have experimented with various *ad hoc* stopping rules for track termination. A simple choice is to terminate a track when N out of M measurements are misassociated. Such rules may be used to approximate the key attribute of an MHT algorithm (e.g., improved report to track association by deferring assignment until sufficient evidence has been accumulated). However, since this is an *ad hoc* expedient, the optimal choice (N, M) as a function of tracking parameters and operating conditions is not clear. Although we have performed initial experimentation, rigorous attention to this issue represents an important future research direction. The goal will be to derive an expression for the PCA suitable for approximating the behavior of an MHT-based tracker.

The PCA expression used in the present study was developed for simplified scenarios for which there is a single class of targets, the noise is Gaussian, and the covariance matrices are identical for all targets. Actual tracking systems are more complex as they use, for example, Variable Structure Interacting Multiple Models (VS-IMM), feature–aided tracking, and context data such as elevation maps and road network information. Current efforts are aimed at extending the PCA expression to cover these cases as well. As a first step, we extended the expression to the case of road–constrained tracking. Since the effect of roads is more difficult to model than the simplified case (which corresponds to off–road tracking), we based the extensions on heuristic considerations. However, we believe that the proposed expression captures key effects associated with road–constrained tracking. In this paper we describe the proposed expression and present results of a simulation to validate these developments.

This paper is organized as follows. Section 2 briefly discusses the data association problem and reviews the analytical PCA expression for the unconstrained (off–road) case. Section 3 describes the TPM while Section 4 presents preliminary results of a validation study comparing the performance of a real tracker with the performance predicted by our model. Section 5 presents the proposed PCA expression for road-constrained tracking, and Section 6 presents results on Monte Carlo simulations that were implemented to validate the extensions of the PCA expression. Section 7 summarizes the results and discusses future work in this area. The Appendix provides a formal discussion on the concept of uncertainty volume as used in this paper.

2. PROBABILITY OF CORRECT ASSOCIATION (PCA)

The PCA expression derived by Mori et. al.[2] is given by

$$P_{CA} = \exp\left(-C_m \beta \det(S)^{1/2} - D_m \beta_c \det(S)^{1/2}\right) \tag{1}$$

where m is the measurement dimension, S is the innovations covariance, β and β_c are the target and false alarm densities, respectively, and C_m and D_m are constants defined as follows:

$$C_m = 2^{m-1}\pi^{(m-1)/2}\frac{\Gamma\left(\frac{m+1}{2}\right)}{\Gamma(m/2+1)}, \tag{2}$$

$$B_m = \pi^{m/2}\frac{1}{\Gamma\left(\frac{m}{2}+1\right)}, \tag{3}$$

$$D_m = B_m 2^{m/2}\frac{\Gamma(m)}{\Gamma(m/2)}. \tag{4}$$

The innovations covariance is given by

$$S = HPH^T + R \tag{5}$$

where P is the KF extrapolated covariance, H is the measurement matrix, and R is the measurement error covariance matrix. In the next section we describe how the PCA expression is used within TPM.

3. TRACKING PERFORMANCE MODEL (TPM)

The objectives of the TPM are to capture essential details of the tracking process, to appropriately draw from random processes in a simulation setting so that robust measures of performance may be computed, and to achieve this in a practical sense. Our approach utilizes two principal assumptions (approximations) to satisfy these objectives. First, in contrast to a high fidelity simulation that lays out and tracks multiple targets, the TPM simulates one target per Monte Carlo run. Second, the impact of multiple targets (for real scenarios) and the performance of the association component of trackers (for actual trackers) is represented via the analytical expression for the PCA. The latter depends on target density, and thereby captures the effect of multiple targets in this way. The additional confusing effect of false alarms is also modeled by the PCA expression. In this way, we predict the ability of a nominal actual tracker to associate the current track with the correct report (with the caveat, as discussed previously, that instantaneous association is assumed). The use of Monte Carlo simulations—instead of a pure analytical study—allows us to account for the variations in the PCA, both statistical and due to kinematics and dynamics of the problem. The causes for these variations include misdetections due to occlusions and minimum-detection-velocity (MDV), variations on the revisit rate, and variations on the sensor/coverage area geometry, among other factors. In addition, misassociations also increase the estimation error covariance, which in turn affects the PCA as well.

A functional block diagram of the TPM is shown in Figure 1. It begins with the initialization of a number of parameters that define the system, including radar performance parameters, platform orbits, tracker settings, cartographic information, and target parameters. A list of these parameters is shown in Table 1. In each iteration of the Monte Carlo run, the initial platform location and the initial location and heading of the target are randomly selected. Both a nominal and true KF covariance matrices are propagated. The *nominal* covariance is the one computed by the KF and is used to determine the KF gain used by the tracker for that particular target. The *truth* covariance matrix is that which results from updating the state estimate with the measurement, which could be either a genuine measurement or a measurement that is a false alarm or belongs to another target and has been misassociated. The next step is to check whether the target is covered by the platform constellation, whether the LOS velocity is larger than the MDV, and whether the target is detected by the radar. To simulate the latter, a detection instance is randomly generated with probability Pd. If all those checks are successful, a positive detection is declared. The probability of correctly associating the detection is evaluated using the PCA expression (1) with S being the truth innovations covariance (as opposed to the nominal one, carried over by the KF). An instance of misassociation is generated with probability PCA. If positive, a

correct association is declared and both the nominal and truth covariances are updated using the KF update equations. If negative, the nominal covariance is updated using the nominal KF update equations, but the truth one is updated assuming that the innovations is a zero mean random variable uncorrelated with the the current state estimate. The rationale is that, given that the incorrect measurement has been associated, the difference between the measurement and the prediction is a random variable with zero mean (conditioned to the current state estimate). The final effect is that the variance of the estimation error decreases if a correct association is made, but the variance increases if the association is incorrect.
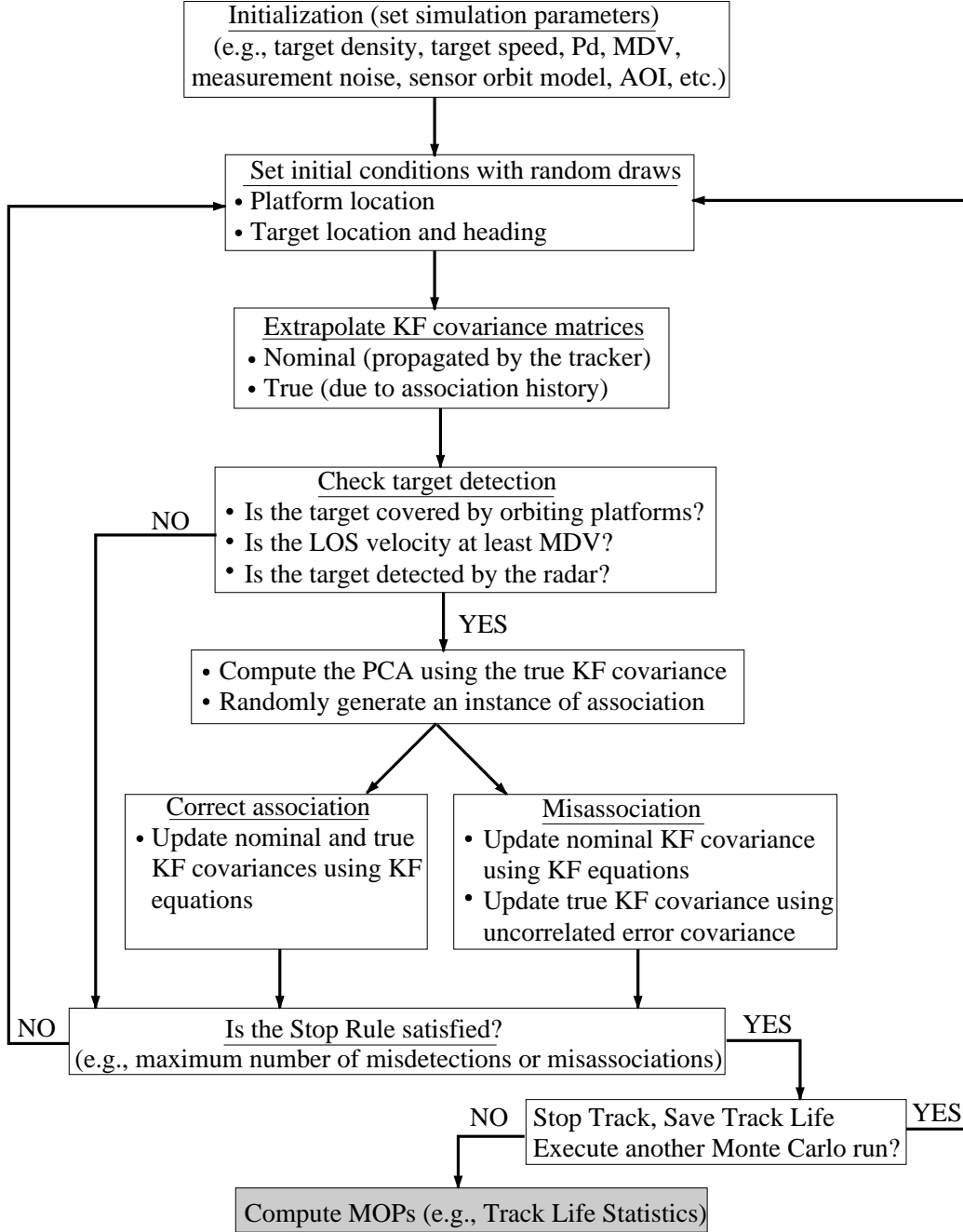

Figure 1. Tracker Performance Model (TPM) functional diagram. The outer loop indicates the iterative cycle of the Monte Carlo simulation.

Table 1. Main TPM parameters.

Radar Performance Parameters	GMTI revisit time
	Minimum grazing angle
	Minimum Detectable Velocity (MDV)
	1-σ Range Error
	1-σ Range Rate Error
	1-σ Cross-Range Error
	Probability of Detection
	False Alarm Density
Tracker Parameters	Promotion and Deletion Rules
	Dimensionality of raw data measurements
Cartographic Parameters	Digital Terrain Elevation Data (DTED) Errors
	Road Network Error
	Road Density
Target Parameters	On–Road Motion Model (Process Noise)
	Off–Road Motion Model (Process Noise)
	Target Density
	Fraction of Targets on roads

The TPM can be used to generate many of the standard tracker MOPs. In our studies thus far, we have focused on the computation of track life statistics. A typical output of the model is cumulative track life probability vs. track life in minutes. To compute these track life statistics, the TPM keeps track of the misdetections and the correct/incorrect associations. As already discussed, it uses an *ad hoc* rule to terminate the track. The rules that we have explored include: (1) one misassociation, (2) two or more misassociations in a row, and (3) N misassociations out of M associations. The first rule assumes that the tracker ends the track as soon as a misassociation occurs. In practice, the tracker is not "cognizant" of that until the accumulation of the errors produce several misdetections. The rule is very conservative as the tracker may recover the track if the next association is correct. Nonetheless, this rule is important because it measures the time in which tracks remain 100% pure, which is a measure of performance *per se*. The second and third rules are somehow more realistic. They assume that the large errors induced by a series of misassociations will trigger enough number of misdetections that induce the tracker to drop the track. It is not clear, however, what values of N and M should be chosen as a function of tracking parameters to represent the effect of track termination. We are currently investigating this effect and will report the results in a future publication.

4. SIMULATION RESULTS

To assess the validity of our TPM, we conducted several simulation experiments using ALPHATECH's Precision Fire Control Tracker (PFCT) to empirically compute MOPs for comparison to those predicted by the TPM. Currently, the TPM only models the basic multi-target tracking functions, and therefore we turned off many of PFCT's capabilities such as Multiple Hypothesis Tracking and feature–aided tracking in order to make a meaningful comparison. In this exercise we ran the TPM with 5 sec revisit time, 0.5 m/s MDV, 6 m range error, 5 m/s range rate error, 300 m cross–range error, 10 m DTED error, and 10 m road network error. The target density was set to $\beta = 5 \; Km^{-2}$, while the false alarm density was negligible. The stopping rule was set to 1 misassociation, so that the track life actually represents the time the tracks remain 100% pure. Following, we generated a scenario for the PFCT that, in average, corresponded to the same parameters. To achieve the same target density, approximately 100 targets where randomly distributed on a disc of radius $R \approx 2500$ m. Initially, the target speed was set to 5 m/s and the target heading was randomly selected. (The target speed can alternatively be drawn from a given probability distribution. By fixing it, the results are given with target speed as a parameter.) The motion of each target was simulated by a motion model with the appropriate random noise. An example display from this process is shown in Figure 2.
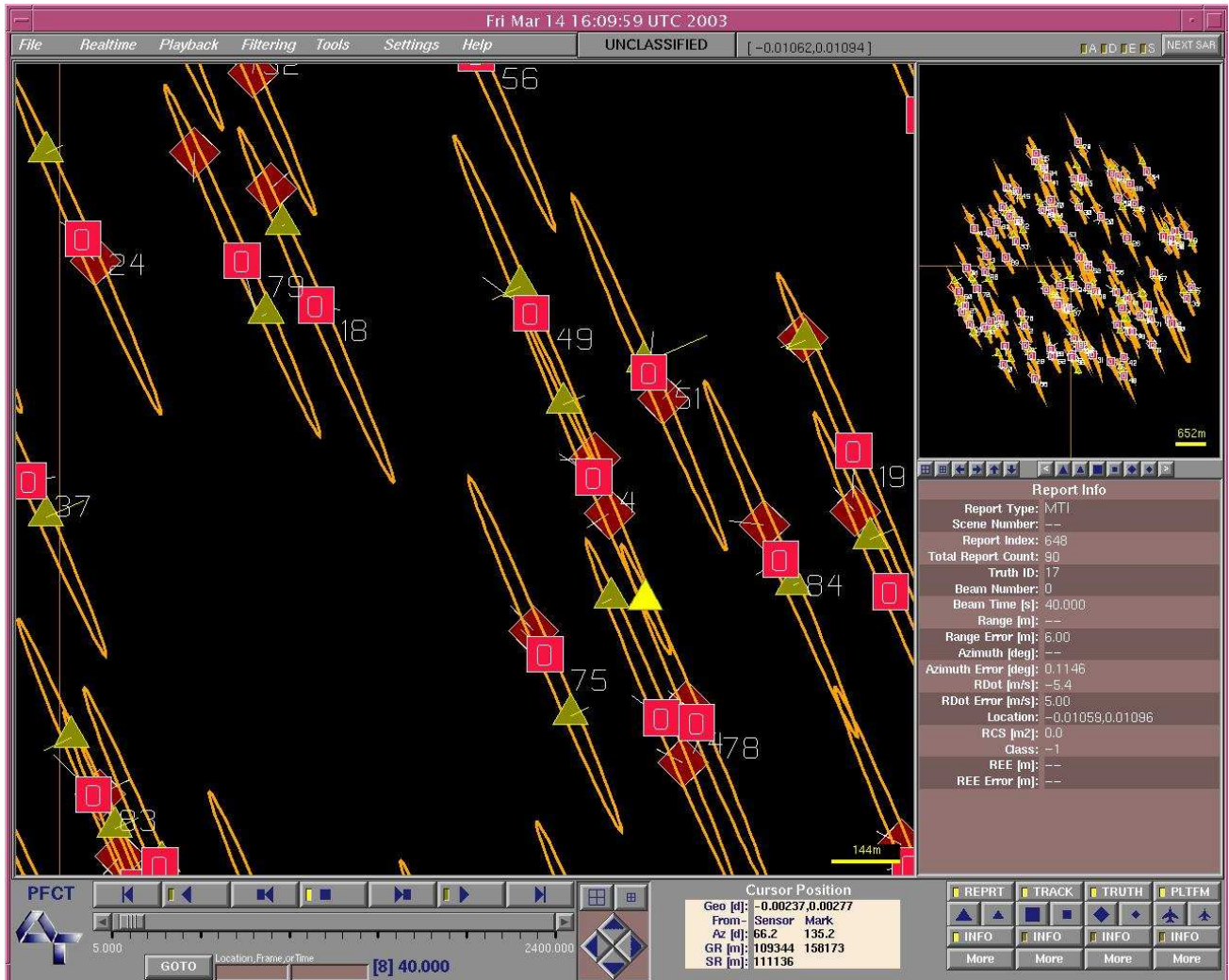
Figure 2. ALPHATECH's Display Manager (DM++) showing the measurements (triangles), the tracks (squares), and the truth (diamonds). The ellipses indicate 1-sigma track errors while the numbers beside the squares are the track IDs. The small screen on the upper right displays all the targets that were laid out on the AOI (which is disc). The main screen is a close–up showing more details. On the bottom, a number of control buttons allow for adjusting the display. On the bottom right, details on either the truth, tracks, or measurements can be displayed as well.

Targets that drift out of the support of the disc have the undesired effect of decreasing the target density. To address this problem, the velocities of such targets were reversed as required. These targets were tagged to be discarded at the time of tracking statistics generation (since the motion of these targets would not correspond to the assigned motion model). The motion of the platform was also simulated (we used a standard racetrack orbit). Sensor reports were then generated and input to the tracker, which generated the standard output consisting of lists of tracks and associated reports. This list was processed along with the list of truth reports to generate track life statistics.

Figure 3 shows the cumulated track life probability vs. track life. These initial results demonstrate a good agreement between the performance plots predicted by TPM and those computed empirically using output from the real tracker. In this example, the stopping rule was set as the time to the first misassociation. We chose this stopping rule for the initial set of experiments to minimize the complexity of the simulation. Additional tests and further simulation experiments are planned.
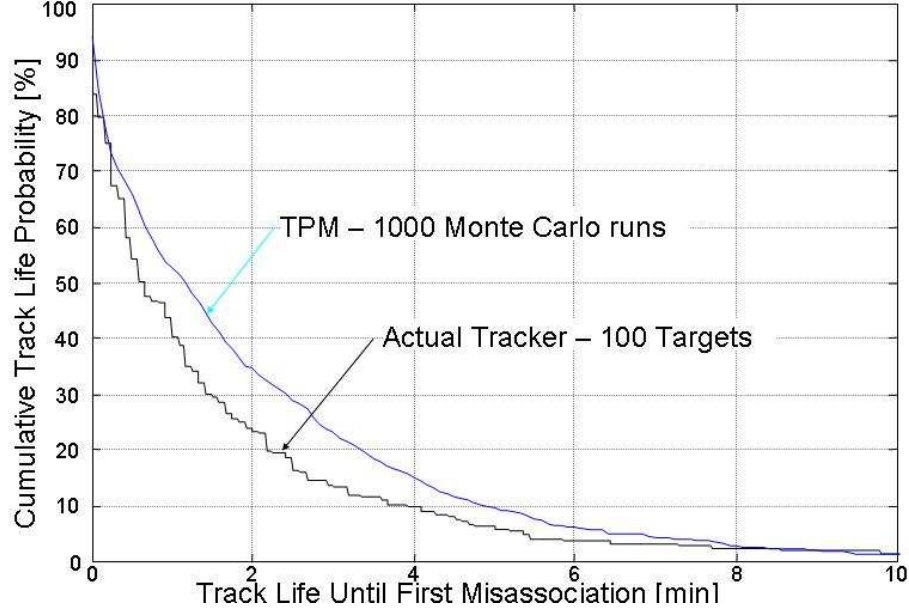
Figure 3. Cumulative track life probability vs. track life for the example displayed in Figure 2.

5. ANALYTICAL EXPRESSION FOR ROAD–CONSTRAINED PCA

Equation (1) can be interpreted as an exponential of the average number of measurements within the "uncertainty volume" of the prediction. To be correctly associated, a given measurement competes with all the measurements—both true targets and false alarms—that fall within the uncertainty volume. Thus, to extend the PCA expression to the road–constrained case, we need to compute the uncertainty volume. The difficulty is that the road–constrained error distribution is not gaussian and depends on the road density. Hence, we formally define the uncertainty volume for arbitrary distributions (see Appendix) and derive an approximate expression for the case of road–constrained tracking.

We begin with preliminary definitions of road density, and one–dimensional and two–dimensional on–road target densities, including normalized versions of these parameters. We then present our expressions for the PCA for off–road and on–road targets and explain the rationale behind these approximations.

Road Density

Road density is defined as the ratio between the total road length within a given area and the area. In terms of the average distance between roads, d, the road density for a grid–like road network is given by:

$$\rho = \frac{\text{total road length within a given area}}{\text{area}} = \frac{2N(Nd)}{(Nd)^2} = \frac{2}{d} \tag{6}$$

Intuitively, the tracking prediction power depends on the ratio between the average distance between roads and the off–road innovations standard deviation, σ_o. Therefore, we define the normalized road density, $\tilde{\rho}$, as follows:

$$\tilde{\rho} = \rho \, \sigma_o \tag{7}$$

The road density is given in terms of "*Km of road per sq Km*". The normalized road density has no units.

On–road Target Density

We define the one–dimensional and the two–dimensional on–road target densities. The one–dimensional density, denoted by λ, is the average number of targets per road length:

$$\lambda = \frac{\text{number of targets on a road}}{\text{length of the road}} \tag{8}$$

7

The two–dimensional density, denoted by β_R, is the average number of on–road targets per area:

$$\beta_R = \frac{\text{number of on–road targets within an area}}{\text{area}} \tag{9}$$

These two densities and the road density defined above are related as follows.

$$\beta_R = \lambda \rho \tag{10}$$

As in the case of off–road targets, the concept of density can be generalized to higher dimensions. A typical three–dimensional density involves the speed of the target as well as the area. In that case, the density is expressed in terms of "number of targets per square Km per m/s."

Road Constrained PCA

The idea is to modify equation (1) to account for the road network. In particular, we will replace the product of C_m and the square root of the determinant of the innovations matrix in (1) by an approximate uncertainty volume. The uncertainty volume is formally defined in the Appendix. The first step is to interpret equation (1) in terms of the average number of targets and false alarms that fall within the uncertainty volume. Notice from equation (22) that for Gaussian distributions and for a given target density, β, the average number of targets within the uncertainty volume is given by

$$\beta V = \beta \det(S)^{1/2} C_m \tag{11}$$

which is exactly the part in the exponent of equation (1) that corresponds to the targets.

For the case of road–constrained tracking, we need to distinguish between targets that are off the road and targets that are on the road. We propose the following heuristic approximations for these two cases:

$$P_{CA|O} = \exp\left(-(\beta_R + \beta_o + \gamma_m \beta_c)V_o\right) \tag{12}$$

and

$$P_{CA|R} = \exp\left(-(\beta_R + \beta_o + \gamma_m \beta_c)V_R\right) \tag{13}$$

where β_o, β_R, and β_c are the off–road, on–road, and false alarm densities, respectively; V_o and V_R are the off–road and on–road uncertainty volumes, and $\gamma_m = D_m/C_m$ is a constant. The rationale for these expressions is that to be correctly associated, measurements compete with targets and false alarms that fall within the uncertainty volume. Thus, if a target is off the road, the average number of measurements that fall in the uncertainty volume is $\beta_R V_o + \beta_o V_o$, while the average number of false alarms is $\beta_c V_o$. The factor γ_m takes into account the fact that random false alarms are more difficult to associate that measurements of other tracks.[1, 2]

To use the PCA expressions (12) and (13) we still need to evaluate V_o and V_R. The off–road uncertainty volume, V_o, is given by the product $C_m \det(S_o)^{1/2}$ because the distribution of the innovations is Gaussian. The innovations covariance matrix, S_o, is computed using the KF equations as in (5). The on–road uncertainty volume, V_R, is more difficult to compute because the distribution is not gaussian and depends on the road density. The correct way to compute it would be to propagate a mix Gaussian distribution and compute the uncertainty volume using equations (18) and (19). Instead, we approximate it based on heuristic considerations. We propose the following approximation:

$$V_R(\rho) = \frac{a + \rho \sigma_o}{b + \rho \sigma_o} V_o \tag{14}$$

where

$$a = b\frac{\sigma_{R_0}^2}{\sigma_o^2} \tag{15}$$

$$b = \frac{\sigma_o^2 - \sigma_{R_1}^2}{\sigma_{R_1}^2 - \sigma_{R_0}^2} \tag{16}$$

The parameter σ_o is the "effective" standard deviation of the target location innovations. By effective we mean the geometric average of the standard deviation in all directions. We define σ_o^2 as the square root of the

determinant of the sub-matrix of S_o that corresponds to the (x, y) components. If the estimates of the (x, y) components are uncorrelated, then $\sigma_o^2 = \sigma_x \sigma_y$ (although the (x, y) components are rarely uncorrelated). The parameters σ_{R_0} and σ_{R_1} are the "effective" innovations standard deviation for the case of $\rho = 0$ and $\rho = 1/\sigma_o$, respectively. We use heuristic expressions for σ_{R_0} and σ_{R_1} as well, since they are difficult to compute.

Equation (14) captures three of the properties of the on–road uncertainty volume that we expect to find:

1. For very large ρ, V_R converges towards V_o, since the road network does not add any information. In this case, $V_R(\infty) = V_o = C_m \sigma_o^2$.

2. For $\rho = 0$, the uncertainty region is given by $V_R(0) = C_m \sigma_{R_0}^2$.

3. For $\rho = 1/\sigma_o$, the uncertainty region is given by $V_R(1/\sigma_o) = C_m \sigma_{R_1}^2$
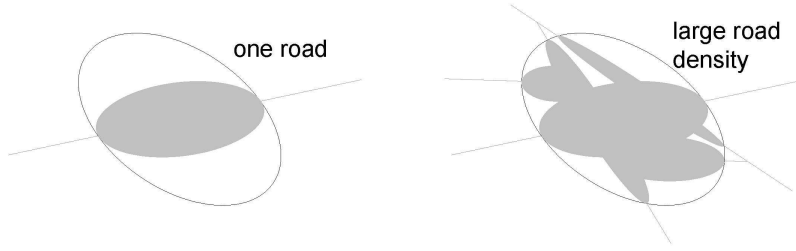
Figures 4 and 5 depict the properties described above.

Figure 4. On–road uncertainty regions (gray areas) for very small density (left) and large road densities (right).
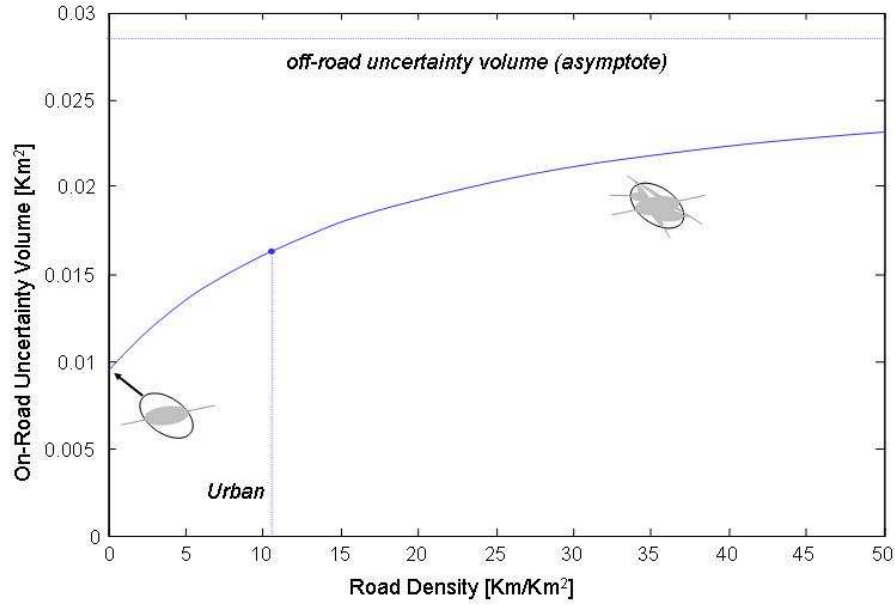

Figure 5. Schematic of the variation of the on–road uncertainty volume with road density.

Average PCA

The average PCA for on–road and off–road targets is obtained by averaging equations (12) and (13):

$$P_{CA} = \frac{\beta_o}{\beta_R + \beta_o} \exp\left(-(\beta_R + \beta_o + \gamma_m \beta_c)V_o\right) + \frac{\beta_R}{\beta_R + \beta_o} \exp\left(-(\beta_R + \beta_o + \gamma_m \beta_c)V_R\right) \tag{17}$$

6. VALIDATION OF THE ROAD-CONSTRAINED PCA EXPRESSION

The derivation of the expression for road-constrained PCA involves several heuristic approximations. Therefore, it needs to be validated against realistic scenarios. Here we report on initial efforts to accomplish this validation. We fixed a basic scenario and varied both the normalized target density and the normalized road density, and computed the PCA using the proposed formula for various on–road/off–road target ratios and various maneuvering ratios (MR). The MR is defined as the ratio between the standard deviation of the position error due to the process noise and past measurements, and the standard deviation of the position error due to the latest measurement. (The MR is similar to the target maneuvering index.[3]) Then we set up experiments to mimic these parameters and validate the PCA expression. In this initial validation effort, we did not use an actual tracker; instead, we followed a similar procedure as in Mori et. al.[2] except for the effect of roads, which was approximated.

The expected total number of objects was set to be 100, keeping the on–road/off–road target ratio set for each experiment. For off–road targets, the prediction error covariance matrix was set to be diagonal with identical terms in the main diagonal. For on–road targets, a random heading was set for every target, and a random prediction error along the road was generated with the off–road prediction variance. The across–road error was generated by drawing a random variable with the off–road prediction variance, and subtracting a random number r_d drawn from a uniform distribution between 0 and d, where d is the average distance between roads in a uniform road network. The resulting across–road error distribution is concentrated around the origin for large d (small road density) and converges towards a Gaussian with the off–road prediction variance for very small d (large road density). Then, we added a measurement noise to both on–road and off–road targets. The ratio between prediction error and measurement noise standard deviations was determined from the maneuvering ratio set for that particular experiment. Finally, we solved the assignment problem using a version of the Munkres algorithm. The PCA was obtained by repeating this experiment 100 times and determining the ratio between the number of correct associations and the total number of associated points. The results are shown in Figures 6 and 7. There is a good agreement between the values obtained by the proposed road–constrained PCA (solid line) and those obtained experimentally. The caveat is that the across–road error probability distribution was generated *ad hoc*. A more comprehensive simulation using a real tracker is planned.
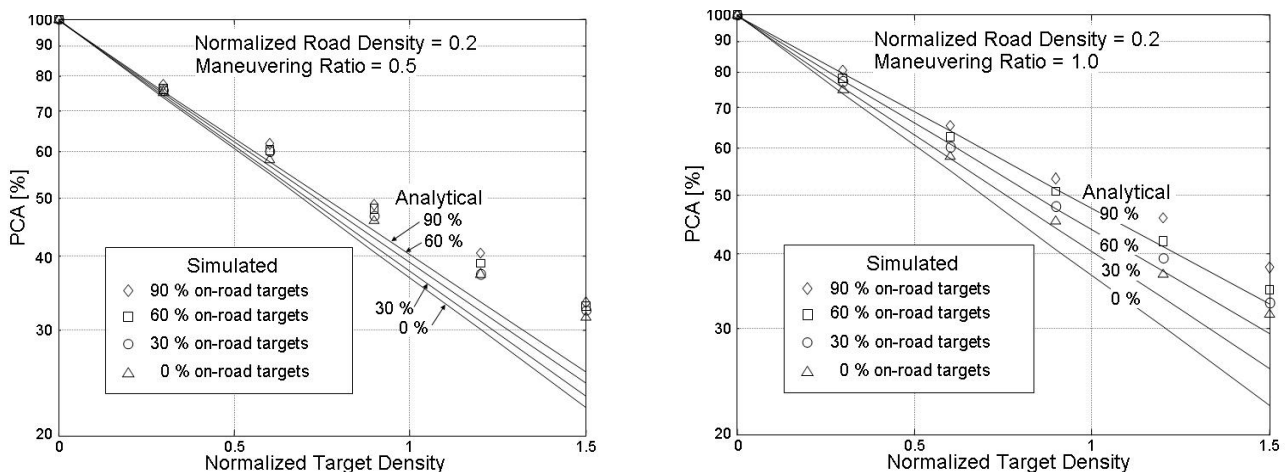

Figure 6. Probability of Correct Association as a function of the Normalized Target Density for various on–road/off–road target ratios and maneuvering ratios, and for a Normalized Road Density of 0.2.

7. CONCLUSIONS

This paper described a Tracker Performance Model (TPM) that predicts the performance of a nominal tracker as a function of a number of tracking parameters. The purpose of this model is to be able to generate approximate measures of performance in a time and cost efficient way. We carried out some preliminary experiments to
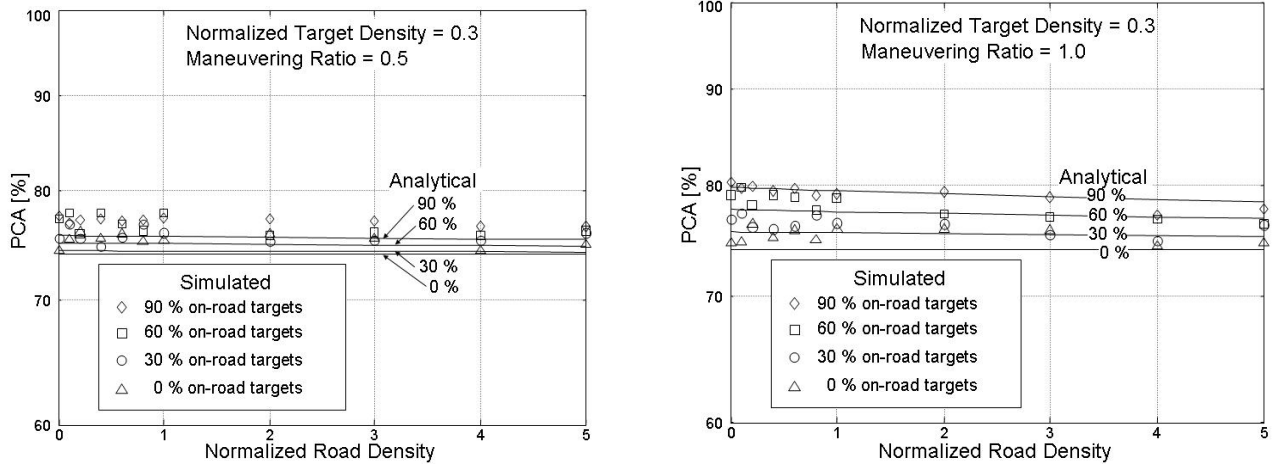
Figure 7. Probability of Correct Association as a function of the Normalized Road Density for various on–road/off–road target ratios and maneuvering ratios, and for a Normalized Target Density of 0.3.

validate TPM and compared the performance of ALPHATECH's Precision Fire Control Tracker (PFCT) against the performance predicted by TPM. Preliminary results show good agreement, what encourages us to continue testing and refining TPM to account for advanced features such as MHT and feature–aided tracking. We also introduced an expression for the PCA for road–constrained tracking and set up simulations to provide some level of confidence on its accuracy. To fully validate the road–constrained PCA expression, however, we will need to experiment with real data or implement a high fidelity Monte Carlo simulation that accurately represents the effect of the roads on tracking performance. We consider this expression as an initial attempt to quantify the effect of roads using a simple analytical expression. Current efforts are aimed at extending the PCA expression to account for MHT explicitly (as opposed to "N out of M" rules) and other effects such as hypothesis tree pruning strategies.

ACKNOWLEDGMENTS

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Appendix: Uncertainty Volume for On-Road Targets

Probability Concentration Region—For arbitrary probability densities and a given percentile, p_m, we formally define the Probability Concentration Region, $\mathcal{R}(p_m)$, as follows:

$$\mathcal{R}(p_m) = \left\{ x \in \Re^m \ : \ P\{x \in \mathcal{R}\} = p_m \text{ and the volume of } \mathcal{R}, \int_{x \in \mathcal{R}(p_m)} dx, \text{ is minimum} \right\} \qquad (18)$$

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Uncertainty Volume—The Uncertainty Volume, V, is defined as the volume of the probability concentration region for a particular percentile:

$$V = \int_{x \in \mathcal{R}(p_m)} dx \tag{19}$$

for

$$p_m = \frac{m\pi^{m/2}}{\Gamma\left(\frac{m}{2}+1\right)} \int_0^{r_m} \frac{\nu^{m-1} e^{-\frac{1}{2}\nu^2}}{(2\pi)^{m/2}} d\nu \tag{20}$$

where

$$r_m = \left(\frac{C_m}{B_m}\right)^{1/m} = \left(\frac{2^{m-1}}{\sqrt{\pi}} \Gamma\left(\frac{m+1}{2}\right)\right)^{1/m} \tag{21}$$

We should point out that this choice of p_m is somehow arbitrary. With this choice the uncertainty volume exactly matches the exponent of the PCA expression, including the constant C_m, but we could have chosen another p_m with similar results. With our choice of p_m, however, the uncertainty volume for Gaussian distributions equals the square root of the determinant of the covariance matrix multiplied by the constant C_m; that is,

$$V = C_m \det(S)^{1/2}. \tag{22}$$

To show this, we evaluate (19) for a Gaussian distribution with mean μ and covariance matrix S. The probability concentration regions are ellipsoids given by

$$\mathcal{R} = \left\{x \in \Re^m \; : \; (x-\mu)^T S^{-1}(x-\mu) \le r\right\} \tag{23}$$

The particular r that corresponds to the percentile p_m is computed as follows:

$$\begin{aligned} p_m &= P\{x \in \mathcal{R}\} \\ &= \int_{(x-\mu)^T S^{-1}(x-\mu) \le r} \frac{1}{(2\pi)^{m/2} \det(S)^{1/2}} e^{-\frac{1}{2}(x-\mu)^T S^{-1}(x-\mu)} dx \\ &= \int_{y^T y \le r} \frac{1}{(2\pi)^{m/2}} e^{-\frac{1}{2}y^T y} dy \\ &= \frac{m\pi^{m/2}}{\Gamma\left(\frac{m}{2}+1\right)} \int_0^r \frac{\nu^{m-1} e^{-\frac{1}{2}\nu^2}}{(2\pi)^{m/2}} d\nu \end{aligned} \tag{24}$$

The last equality is obtained by using the following property of spherical integrals[2]:

$$\int_{\|y\| \le r} f(\|y\|) dy = mB_m \int_0^r \nu^{m-1} f(\nu) d\nu \tag{25}$$

By comparing equations (24) and (20), it follows that r is in fact the r_m given by equation (21). Now, the uncertainty volume for this probability concentration region becomes:

$$\begin{aligned} V &= \int_{(x-\mu)^T S^{-1}(x-\mu) \le r_m} dx \\ &= \det(S)^{1/2} \int_{y^T y \le r_m} dy \\ &= \det(S)^{1/2} mB_m \int_0^{r_m} \nu^{m-1} d\nu \\ &= \det(S)^{1/2} B_m r_m^m \\ &= \det(S)^{1/2} C_m \end{aligned}$$

where again we made use of the expression (25).